12

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Effect of Electron Spin Paramagnetism on the Velocity of Sound in Metals

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18214 A

By means of a simple calculation it is shown that the interaction of an external magnetic field with the magnetic moment of the conduction electrons in a metal gives rise to an increase in the velocity of sound which is independent of the angle between the direction of the magnetic field and the direction of propagation of the sound wave.

Author

THE effect of magnetic fields on the velocity of sound has been studied recently by many authors.[1] The theory predicts an increase in the velocity of sound which depends quadratically on the magnetic field and depends also on the angle between the magnetic field and the direction of propagation of the sound wave. Furthermore, de Haas-van Alphen type oscillations are predicted in the magnetic field dependence. All these effects have been confirmed experimentally.[2] In this paper we wish to show that the interaction of the magnetic field with the magnetic moment of the conduction electrons also leads to a finite though small increase of the velocity of sound. However, as we will see, this effect is independent of the angle between the direction of the magnetic field and the direction of sound propagation. Since all effects of magnetic fields on the velocity of sound appear through small changes in the dielectric behavior of the conduction electrons, all such changes are additive. Therefore, even for the highest attainable fields, we may concentrate exclusively on the magnetic moment interaction and merely add all previously calculated effects to our final result. We start with the well-known dispersion relation for longitudinal phonons in the random phase approximation (RPA)[3]

$$\omega^2=\omega_i^2/\epsilon(K,\omega)\,, \tag{1}$$

where ω_i is the ionic plasma frequency and ϵ the dielectric constant in the RPA[4]:

$$\epsilon=1-\frac{e^2}{\pi^2 m}\int d^3k\frac{F(k)}{[\omega-(h/m)\mathbf{k}\cdot\mathbf{K}]^2-[(h/2m)K^2]^2}. \tag{2}$$

The Fermi distribution at 0°K is simply given by a step function:

$$F(k)=S(k_F-k)\,, \tag{3}$$

with $k_F=(3\pi^2 N)^{1/3}$ the Fermi momentum divided by $\hbar$. Upon application of a uniform constant magnetic field in, say, the z direction the magnetic moment interaction $-\mu\boldsymbol{\sigma}\cdot\mathbf{H}$ (the Pauli term) induces a change in the Fermi distribution. For electrons with spin up, it becomes

$$F_+(k)=S\{[k_F^2+(2m\mu/h^2)H]^{1/2}-k\} \tag{4}$$

and for electrons with spin down

$$F_-(k)=S\{[k_F^2-(2m\mu/h^2)H]^{1/2}-k\}\,. \tag{5}$$

Here μ is the magnetic moment of an electron and H is the magnitude of the applied magnetic field. All we have to do now is to replace $F(k)$ in expression (2) by $\frac{1}{2}F_+(k)+\frac{1}{2}F_-(k)$ in order to obtain the proper dielectric constant in a magnetic field. Of course we did not take into account the influence of the magnetic field on the orbital motion of the electrons. This has been done by Quinn and Rodriguez.[1] But as we mentioned earlier the effects, being small, are simply additive. Substituting then Eqs. (4) and (5) into expression (2) and expanding in powers of H we find to lowest order in H:

$$\epsilon(k,\omega)=1+\frac{3\omega_P^2}{v_F^2K^2}-\frac{2e^2\mu^2H^2}{\pi\hbar^3 v_F^3K^2}. \tag{6}$$

Equation (6) is valid in the long-wavelength limit. We also neglected terms of the order of c_s/v_F, c_s being the velocity of sound and v_F the Fermi velocity, these terms being of the order of 10^{-3}. Inserting Eq. (6) into Eq. (1) we finally obtain:

$$c_s'=c_s[1+(e^2\mu^2/3\pi\hbar^3v_F\omega_P^2)H^2]\,, \tag{7}$$

where

$$c_s=(m/3ZM)^{1/2}v_F\,, \tag{8}$$

with Z the number of conduction electrons per atom. Equation (8) is the well-known Bohm-Staver result for the velocity of sound.[3] That the effect is quite small may be seen by actually inserting numbers. It turns out, for instance, that for potassium

$$(c_s'-c_s)/c_s=4.3\times10^{-19}H^2. \tag{9}$$

A change in the velocity of sound of 1 part in 10^7 can be measured at present.[5] A change of this magnitude

[1] J. J. Quinn and S. Rodriguez, Phys. Rev. Letters **9**, 145 (1962); M. J. Harrison, Phys. Rev. Letters **9**, 299 (1962); M. H. Cohen, M. J. Harrison, and W. A. Harrison, Phys. Rev. **117**, 937 (1960); G. A. Alers and R. T. Swim, Phys. Rev. Letters **11**, 72 (1963).
[2] G. A. Allers and R. T. Swim, Ref. 1.
[3] D. Bohm and T. Staver, Phys. Rev. **84**, 836 (1952).
[4] J. Lindhart, Kgl. Danske Videnskab. Mat. Fys. Medd. **28**, No. 8, 1 (1954).
[5] R. L. Forgacs, IRE Trans. Instr. **19**, 359 (1960).

requires a field of 4.8×10^5 G according to (9). Fields of this strength are presently unavailable. Therefore, an increase in accuracy of measurement is required to make the effect observable. For transition metals with their high paramagnetic susceptibilities we expect a much larger effect, however. Owing to their nonspherical Fermi surfaces, the effect will depend on crystal orientation.

As opposed to the other effects calculated previously,[1] Eq. (7) shows that the effect considered here is independent of the direction of propagation of sound wave.